Constrained Optimization

Lecture 47
Section 7.5

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Objectives

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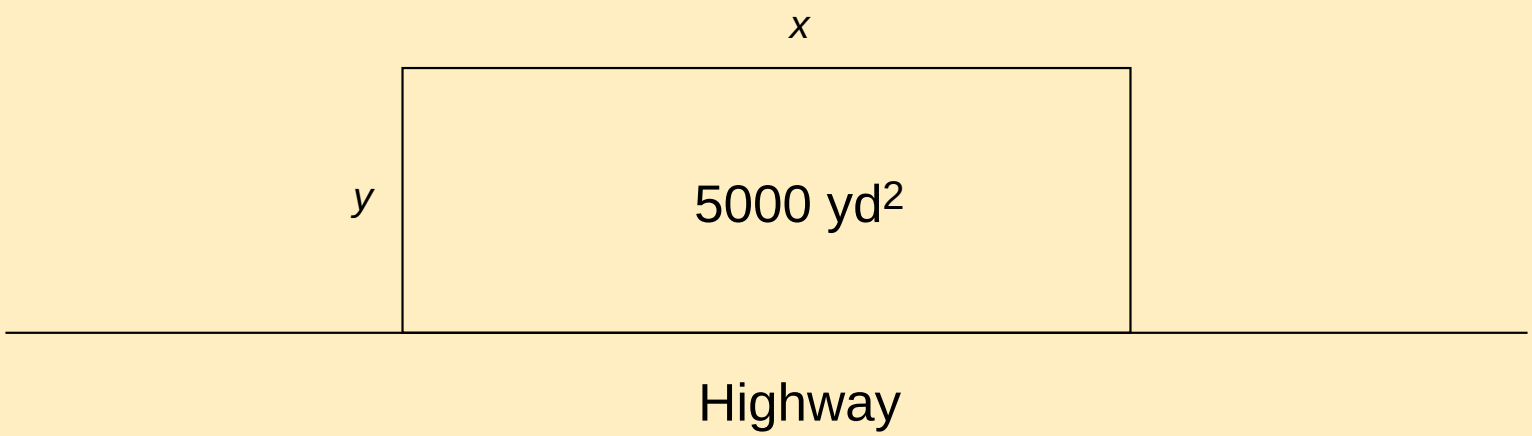
- Learn the method of Lagrange multipliers.
- Apply the method.

An Example

Example 7.5.1:

The highway department is planning to build a picnic area for motorists along a major highway. It is to be rectangular with an area of 5,000 square yards and is to be fenced off on the three sides, but not on the side adjacent to the highway. What is the least amount of fencing that will be needed to complete the job?



Constraints

Constraints

- The previous example had a **function to minimize**:

$$f(x, y) = x + 2y.$$

- It also had a **constraint**:

$$xy = 5000.$$

The Method of Lagrange Multipliers

(1) Given the function $f(x, y)$ to be optimized and the contraint $g(x, y) = k$ to be imposed, create the function

$$F(x, y, \lambda) = f(x, y) - \lambda(g(x, y) - k).$$

(2) Solve the system of equations

$$\begin{aligned} F_x(x, y, \lambda) &= 0, \\ F_y(x, y, \lambda) &= 0, \\ F_\lambda(x, y, \lambda) &= 0. \end{aligned}$$

λ is the (Lagrange) multiplier.

An Example

Example:

The highway department crew needed 200 yards of fencing to enclose 5000 square yards, but when they reached the site of the picnic area, discovered that they had only 160 yards. So they decide to fence in the largest picnic area possible with 160 yards of fencing. What is the greatest amount of area they can enclose with 160 yards of fencing?

An Example

Example:

You have \$50 to spend on your girlfriend's birthday present. You can by her candy or jewelry or any combination of candy and jewelry, but you want score as many "points" with her as possible. Let x be the amount spent on candy and y the amount spent on jewelry. You estimate that you will earn

$$C(x) = 500 - \frac{500}{x+1}$$

points for spending x dollars on candy and

$$J(y) = 2y^2$$

points for spending y dollars on jewelry. How much should you spend on each in order to maximize the number of points earned?